THE MATHEMATICAL GAZETTE

EDITED BY

W. J. GREENSTREET, M.A.

WITH THE CO-OPERATION OF

F. S. MACAULAY, M.A., D.Sc.

AND

PROF. E. T. WHITTAKER, M.A., F.R.S.

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If Copies of the Gazette fail to reach a member in consequence of a change of address, of which the Secretaries have not been informed, duplicate copies cannot be supplied.

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NOTICE.

The following Reports have been issued by the Association:—(i) "Revised Report on the Teaching of Elementary Algebra and Numerical Trigonometry" (1911), price 3d. net; (ii) "Report on the Correlation of Mathematical and Science Teaching," by a Joint Committee of the Mathematical Association and the Association of Public School Science Masters, price 6d. net; (iii) A General Mathematical Syllabus for Non-Specialists in Public Schools, price 2d. net.; (iv) Report on the Teaching of Mathematics in Preparatory Schools, 1907, price 3d. net. These reports may be obtained from the Publishers of the *Gazette*.

(v) **Catalogue of current Mathematical Journals**, with the names of the Libraries where they may be found. Pp. 40. Price, 2s. 6d. net.

"Even a superficial study convinces the reader of the general completeness of the catalogue, and of the marvellous care and labour which have gone to its compilation."—*Science Progress*, Jan. 1916.

(vi) **Report of the Girls' Schools Committee, 1916**: Elementary Mathematics in Girls' Schools. Pp. 26. 1s. net.

(vii) **Report on the Teaching of Mechanics, 1918** (*Mathematical Gazette*, No. 137). 1s. 6d. net.

(viii) **Report on the Teaching of Mathematics in Public and Secondary Schools, 1919** (*Mathematical Gazette*, No. 143). 2s. net.

HON. SECRETARIES OF BRANCHES OF THE MATHEMATICAL ASSOCIATION.

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	Mr. G. Goodwill, 37 Park Road, Dulwich, S.E. 21.
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	Miss M. L. Tanner, Broughton High School.

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VOL. XI. MARCH, 1922. No. 157.

The Mathematical Association.

President: THE REV. CANON J. M. WILSON, D.D.

THE Annual Meeting of the Mathematical Association was held at the London Day Training College, Southampton Row, London, W.C. 1, on Monday, 2nd January, 1922, at 5.30 p.m., and Tuesday, 3rd January, 1922, at 10 a.m. and 2.30 p.m.

MONDAY EVENING, 5.30 *p.m.*

Sir George Greenhill, F.R.S., delivered an address on "Mathematics in Artillery"—Before and After the War: A Review of the Outlook: Then and Now.

TUESDAY MORNING, 10 *a.m.*

BUSINESS.

(1) The following Report of the Council for the year 1921 was distributed and taken as read:

REPORT OF THE COUNCIL FOR THE YEAR 1921.

DURING the year 1921, 92 new members have been elected, and the number of members now on the Roll is 832. Of these, 8 are honorary members, 48 are life members by composition, 25 are life members under the old rule and 751 are ordinary members. The number of associates connected with Local Branches of the Association is about 365.

The Council regret to have to record the deaths of Miss E. L. Clarke, of Tunbridge Wells High School; Professor R. B. Clifton, F.R.S., who had been an honorary member of the Association since January, 1882, when the first honorary members were elected; Mr. H. T. Gerrans, Fellow and Tutor of Worcester College, Oxford; the Rev. J. B. Lock, Fellow and Bursar of Caius College, Cambridge,

who was one of the Honorary Secretaries of the Association for the year 1884; Mr. J. R. Marrack, of Tiverton; the Right Honourable Lord Moulton of Bank, G.B.E., K.C.B., F.R.S., who was an original member of the Association in 1871, and held the office of President from 1901 to 1903; and Mr. J. Walmsley, who had been a member of the Association for 40 years and headmaster of Eccles Grammar School for 48 years.

It was hoped that a summer meeting would be held in September in Edinburgh in conjunction with the Edinburgh Mathematical Society and the British Association, but eventually the arrangements fell through.

The Council have had under consideration the defects of the existing scheme for the election of the Teaching Committees. A new scheme will be prepared, and will be submitted for the approval of the members of the Association at the annual meeting in January, 1923.

The Rev. Canon J. M. Wilson, D.D., retires at this meeting from the office of President. He was one of the founders of the Association for the Improvement of Geometrical Teaching in 1871, and in January, 1921, the 50th anniversary of the foundation, he honoured this Association by becoming its President. At the last annual meeting he gave the Association his very interesting and valuable reminiscences of the birth and the early years of the Association, and enabled the Association to put on record the history of its youth. Canon Wilson does not wish to hold office for a second year; and the Council accept his resignation with great regret and with grateful thanks to him for the unique service which he rendered to the Association on the occasion of its Jubilee.

As Canon Wilson's successor the Council have the pleasure of nominating Sir T. L. Heath, K.C.B., K.C.V.O., Sc.D., F.R.S., to be President for the years 1921 and 1922. They also nominate Canon Wilson to be a Vice-President.

Professor W. P. Milne and Principal J. L. S. Hatton retire now from the Council by rotation, and are not eligible for re-election for the coming year. The members present at the Annual Meeting will be asked to nominate and elect others to fill the vacancies.

Mr. C. E. Williams, who has held the office of Librarian since 1908, wishes now to retire. The Council accept his resignation with great regret, and feel that the members would wish to express to Mr. Williams their high appreciation of his services to the Association in the management of the Library during the last twelve years.

In the course of the present month the Library will be removed to the premises of the Incorporated Association of Assistant Masters at 29 Gordon Square, W.C. 1.

The Council again desire to acknowledge the indebtedness of the Association to Mr. Greenstreet for his services as Editor of the *Mathematical Gazette*, and to offer their thanks to the authorities

of the London Day Training College for their kindness in affording accommodation for the Annual Meeting, and for the meetings of the Council and the Committees which have been held during the year.

(2) The Treasurer's Report for the year 1921 was presented and adopted.

(3) The following Reports of the Teaching Committees were read:

REPORT OF THE TEACHING COMMITTEE FOR THE YEAR 1921.

THE General Committee has had only one meeting during the year; the Executive Committee has met twice, and postal votes of the whole committee have been taken on three occasions.

At the meeting of the General Committee on 3rd January, 1921, the chief matters discussed were (1) London Matriculation, (2) the Board of Education Questionnaire on the educable capacity of boys and girls, (3) the draft Mathematical Syllabus for the proposed examination for the recruitment of the Clerical Class of the Civil Service, (4) the revision or reprinting of the Mathematical Association reports on the Teaching of Mathematics.

A new mathematical syllabus for London Matriculation has been issued during the past year which incorporates many of the suggestions of this Association, in particular the use of logarithms is allowed in the Arithmetic and Algebra paper, the use of the properties of similar figures is allowed in the Geometry paper, and Elementary Calculus is allowed as an alternative in the more advanced paper.

A special sub-committee drew up a reply to the Board of Education Questionnaire, the reply being based on letters received from the other members of the Committee.

The Committee made many suggestions with a view to improving the Clerical Class syllabus, and these suggestions were duly acknowledged by the Civil Service Commissioners. In their reply this Committee expressed regret that the Commissioners should issue an entirely new syllabus differing in many respects from the syllabus of the First School Examinations. The Committee is still negotiating with the C.S. Commissioner and the various Examining bodies with a view to securing some degree of uniformity in the syllabus.

With regard to the M.A. reports the Committee decided that it was not advisable at present to revise any of the Reports, but that they were well worth reprinting.

Two letters were received during the year criticising certain Examination papers, and asking the opinion of the Committee. One of these referred to the Arithmetic papers set by the Irish Intermediate Board in 1920. The Executive Committee considered this letter fully, agreed with the censure and forwarded their views to the writer of the letter. The second letter complained of the Geometry paper set at the London General School Examination last summer; this letter is still under consideration.

REPORT OF THE GIRLS' PUBLIC SCHOOLS COMMITTEE FOR THE YEAR 1921.

THE Girls' Schools Committee has met four times during the year. At the request of the General Teaching Committee it reported on the Examination papers of the Intermediate Board of Ireland and on a Questionnaire relating to the educable capacity of boys and girls. Entrance Scholarship Examinations and advanced courses have also been discussed. The Committee is at present considering the question of the teaching of Mechanics in girls' schools.

M. J. GRIFFITH, *Hon. Sec.*

(4) The Election of Officers and Council for the year 1922 was then proceeded with.

Sir T. L. Heath, K.C.B., K.C.V.O., D.Sc., F.R.S., was elected President of the Association for the years 1922 and 1923 in succession to the Rev. Canon J. M. Wilson, D.D.

The Rev. Canon J. M. Wilson, D.D., retiring President, was elected a Vice-President of the Association.

Mr. A. Dakin, Headmaster of Stretford Secondary School, Manchester, and Mr. C. E. Williams were elected Members of the Council in succession to Principal J. L. S. Hatton and Prof. W. P. Milne, D.Sc., who retired by rotation.

Mr. W. E. Paterson was elected Librarian in succession to Mr. C. E. Williams.

The following papers were read in the course of the meeting.

GLEANINGS FAR AND NEAR.

101. That is one great point gained, as Mr. Weaver, a travelling astronomical lecturer, who carried the universe about in a box, told us. "Sir," he said to my father, "when you look at a map, do you know that the east is always on your right hand, and the west on your left ?"—"Yes," replied my father, with a very modest look, "I believe I do." "Well," said the man of learning, "that's one great point gained."—Maria Edgeworth to Miss Ruxton, 1794.

102. "The student may ask, How can anything but *an angle* have a sine ? I answer, that θ is not an *angle*, but the *number* of arcual units in an angle. Every *number* has a *sine*."—De Morgan's *Trig. and Double Alg.* p. 37, 1849.

103. The only other literary work, which the Reminiscent has begun and left unfinished, was a *History of the Binomial Theorem ;* for he too has had his algebraic hours and disported with imaginary quantities ; but he found the allurements of these so strong, as to make it absolutely necessary . . . to divorce himself entirely from them. Perhaps the reasoning on impossible quantities and exterminating them by algebraic operations, till the impossible symbols disappear, and an equation of such quantities is produced, is the highest and most delightful effort of the human understanding ; but its hold on the mind makes it absolutely incompatible with professional duty. The writer was therefore obliged to abandon it : "Et multum formosa vale !" was his exclamation, when he parted from algebra and consigned his binomial lucubrations to the flames.

If the Reminiscent were desired to mention the moment of his literary life in which he experienced the greatest literary delight, he should, without hesitation, mention that in which, for the first time, he perused the first problem of Euclid, and saw the new world of intellectual gratification which was opened for him . . . .

Footnote.—The profound and extensive classical knowledge of the late Mr. Porson is well known ; his knowledge also of algebra and geometry was respectable. He had meditated a new edition of Diophantus, and an illustration of it by the modern discoveries. A short time before he died he gave the Reminiscent an algebraic problem, which, though not of the highest order, is certainly curious. We suppose some of our readers may wish to see it ; we therefore insert it, and our solution of it, in the appendix.—[*Reminiscences of Charles Butler, Esq.*, of Lincoln's Inn, 1824, 4th ed., p. 239. Alban Butler (*Lives of Saints*) was Charles Butler's paternal uncle.]

THE STRUCTURE OF THE ATOM.

By J. W. Nicholson, M.A., D.Sc., F.R.S.

Had I been called upon to address the Association a few years earlier on a subject of this nature, my task would have been much more difficult. Model atoms were then becoming almost as numerous as the sands of the sea. The very simplicity of the quantitative phenomena shown by the simplest atoms was an obstacle to progress. For example, the simple spectrum of Hydrogen,—the so-called Balmer series,—consisting of a set of frequencies given by the formula

$$\nu = N\left(\frac{1}{n^2} - \frac{1}{m^2}\right),$$

where n and m take all possible integral values, was somewhat too simple if considered by itself instead of in relation to phenomena of a widely different type. And as a consequence, we had at least five completely different forms of suggested structure of a Hydrogen atom, each of which, by suitable mathematical artifices, could be made to yield such a set of frequencies.

The main secret of recent progress is our improved knowledge of the nature of positive electricity and of its function in the atom. We could never hope that speculation alone would provide such knowledge, practically all of which has indeed come from the experimental side. We may regard it as a demonstrated fact that positive electricity exists in units even smaller in volume than the electron or unit of negative electricity. But while their volume is smaller, their mass is very much greater, and a unit of positive electricity, the charge $+e$, is 1835 times as massive as the negative unit $-e$, whereas its radius is correspondingly smaller, the mass of a charge of electricity being *inversely* as its radius a, according to the formula

$$m = \frac{2}{3}\frac{e^2}{ac^2},$$

where c is the velocity of light.

As in mechanics, when we probe into its inner meaning, we find that mass is a property which only makes itself felt in relation to change of motion, so also in electrical theory. Students of hydrodynamics are familiar with the fact that a frictionless fluid does not resist the steady motion of a body travelling through it, but nevertheless gives it an apparently greater mass or inertia, which is manifested in any attempt to stop the body or to increase its speed. This extra inertia is a mathematical device which gives a convenient mathematical summary of at least some of the main effects due to the extra motion set up in the surrounding fluid, which involves a supply of energy to the mass for communication to the fluid.

In these days, we know that all mass is essentially of an analogous nature,—that is to say, it is a tangible sign of the existence of kinetic energy, and in many important respects, a measure of the quantity of such energy. But it would be untrue to say that the energy in any sense resides in the limited region of space occupied by the body to which we credit the mass.

The laws of electromagnetic theory in no way resemble, in their mathematical form, those of the motions we deal with in Particle or Rigid Dynamics, but nevertheless, when an electron is set into motion, the whole surrounding aether, extending to infinity, is moved also,—I shall speak in terms of an aether, and crave indulgence from any "relativists" who prefer other forms of words. The necessary energy must be supplied, and one of its manifestations is in the endowment of the electron with a mass. But the law of concentration of the energy near the electron is such that a diminution of its radius increases the necessary energy, so that the smaller the electron, the more inert it would be. Moreover, the mass depends on the velocity which the

electron has at the moment we endeavour to accelerate it, the mass being, when it has a velocity v,

$$\frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}},$$

where m_0 is the "mass for slow speeds," or the quantity $\frac{2}{3}\frac{e^2}{ac^2}$ mentioned already. This is the so-called Lorentz electron, which has finally replaced all others,—in which the variation with velocity was different,—on the double ground that its mass formula is in accord with accurate experiments of Kaufmann and others on the mass of very rapidly moving β-particles, or electrons with a velocity approaching that of light, and secondly that it fits into the formulae of the Principle of Relativity.

If $v=c$, the mass is infinite, so that no finite force can set an electron into motion with the velocity of light. There is general agreement now that all mass is of this ultimate electromagnetic nature, and a manifestation of energy residing in the aether.

Why is the charge on every electron, and the radius and therefore the mass of every electron with a given speed, the same? All are absolute constants of nature. Personally I prefer to regard the "radius" of an electron as a line-constant which pertains to the whole aether, and therefore appears in its energy content always,—in other words, the medium has a structure, and an electron is a region in which a complex knot is tied in the structure.

But perhaps I have said enough about the electron. There is little more to add about the positive unit of electricity, which is the other essential constituent of the atom. For the majority of purposes, the positive unit differs only in being positive and not negative, but it is more concentrated and therefore of greater mass,—signifying perhaps another line-constant in the aetherial structure, of smaller magnitude, in which another kind of knot has been tied.

Most of the historic model atoms have disappeared in the light of recent experimental work, which has shown us that the positive unit is so small,— and further, that it still continues to attract the negative according to the inverse square law of distance, even at distances of only atomic size,—which means about 10^{-8} centimetres at most. The radius of the electron is of as low an order as 10^{-11}, or a thousand times smaller, and that of the positive unit is about 2000 times smaller still. If they do not move too fast, they are quite effectively, even inside the atom, *particles* of electricity rather than collections of electricity having appreciable extension. In fact, the dynamics of atoms accordingly presents more resemblance to what teachers call "Particle Dynamics" than to "Dynamics of Rigid Bodies."

An atom of matter consists of positive units and electrons, the former being packed together,—perhaps bound by other electrons, in a core or nucleus of an atom, of very small size. For an ordinary atom without an electrical charge, the excess of positive electricity in the nucleus is balanced by orbital motions of an equivalent number of electrons, the whole atom being thus electrically neutral. Apparently the only thing we can alter by outside agencies, such as great heat, is the group of "outer" electrons, which may lose one or more, add to their number, or change their orbital configurations. The main problem of atomic structure to-day is to determine these orbits and the conditions which secure their permanency.

There is a curious indefiniteness about an atom thus defined, which we may illustrate by a consideration of a simple Hydrogen atom, now known to contain only one positive unit and one electron. The positive has such a mass that we may regard the electron, with its mobility, as describing an orbit about it, and this orbit may be circular. Under the inverse square law, only

one equation connects the radius a and the angular velocity ω, which is the expression

$$a^3\omega^2 = \text{const.}$$

of Kepler's law. We cannot determine a and ω,—in dynamics we should require "initial conditions," which in a congeries of impacts in a vacuum tube might be anything. In fact, all circles seem possible, with appropriate values of ω. And yet ω must have only a set of possible values not in a continuous sequence, or the spectrum of the atom could not be a set of discrete frequencies. Clearly a new universal constant of nature, not hitherto appreciated, exists as a second determining feature in place of "initial conditions," for there is an identity of form among Hydrogen atoms, whatever their previous history. For some reason, previous history is not operative in the configurations and internal motions of atoms. This is the first fundamental difficulty which retards any glimpse of the meaning of the phenomena of spectra, always regarded even on the quantitative side, as the most conclusive test of any model. For spectra can be measured with an accuracy of five significant figures, and even more. Other atomic properties admit no such accuracy of measurement.

Another very fundamental difficulty was once pointed out by the late Lord Rayleigh. Even spectra not so simple as the Balmer series of Hydrogen frequencies clearly follow a generalisation to which the next approximation is

$$\nu = N\left\{\frac{1}{(m+\mu)^2} - \frac{1}{(n+\mu')^2}\right\},$$

where μ, μ' are constant, N is a *universal* constant belonging to all line spectra, and m and n take all integer values. This in fact is Rydberg's formula. It appears naturally as a formula for ν, and not for ν^2. Now, if spectra are regarded, as till recently, as a manifestation of dynamical vibration of the atomic constituents about their steady motions, we cannot, except in one way, obtain formulae for ν, but always for ν^2, which do not involve square roots. I do not wish to dwell on this, but it is clear to spectroscopists that a correct theory must give values directly for ν and not ν^2.

Again, there are an infinite number of lines, at least theoretically, involved in Rydberg's formula, and a very finite number of dynamical degrees of freedom of the atomic constituents. It is clear that the supposition that frequencies in spectra are frequencies of atomic vibrations, must be abandoned.

The one exceptional case mentioned above occurs when we postulate large magnetic forces of certain types in the atom, as in Ritz' model. Then we obtain ν, and can in fact obtain a working model, by mathematical artifices. But it is entirely contradicted by the more modern experimental evidence.

At this point enters the Quantum theory, which alone has made further progress possible, and there is no longer any room for doubt that, in its main essentials, there is truth in the theory, and especially in its application to the elucidation of atomic structure by means of spectra,—incomprehensible as the theory may be. Into the origin of this theory in the hands of Planck, there is no time to enter, but it arose in a way quite remote from the applications I am about to mention. Its essential feature, to which considerable circumstantial proof was attached by other physical phenomena, is that ultimate radiating systems do not radiate energy continuously, but in spurts or quanta, whose magnitude depends on a new universal constant h of nature, equal to $6{\cdot}55 \times 10^{-27}$ C.G.S. units. Such a packet of energy exhibits itself with a frequency, say ν, such that $h\nu$ is the whole amount of energy in the packet. It is, of course, extremely small, and it is only when the frequency becomes very large that we can expect this discontinuity of emission to produce effects distinguishable from those of completely continuous emission of really infinitesimal elements of energy. But the reason for its choice of the frequency it will show is entirely wrapped in mystery, and it remains as one of the cardinal problems, if not the cardinal problem of all, in the theory of radiation.

We see that h is a quotient of energy and frequency, or a product of energy and time,—an *action*, as the term is understood in higher dynamics. The concept of action, as a mathematical entity, was well-known before, as in the Principle of Least Action, on which all dynamics can take its stand. There is thus apparently a universal constant of action. I proposed once to regard this constant not as one of action, but of *angular momentum*,—a concept more readily understood, and not differing in type. For action and angular momentum have the same dimensions, as have kinetic energy and work. And as a fundamental object was to obtain a new determining equation for rotation of electrons in an atom, the angular momentum appeared a likely quantity to be predetermined. With the supposition that it could have a series of discrete values, an account was found possible of certain spectra,—simpler than series spectra and of a different type,—found from the nebulae and solar corona, and subsequently it appeared that these discrete values were closely related to h.

Bohr subsequently postulated that the angular momentum was always a multiple of $\dfrac{h}{2\pi}$, and gave a remarkable account of the Hydrogen spectrum on this basis. We may recapitulate the main points. If an electron of charge e describes a circle of radius a with angular velocity ω, there is a Kepler relation

$$ma\omega^2 = \frac{e^2}{a^2}.$$

If we add the angular momentum specification,

$$ma^2\omega = \frac{\tau h}{2\pi},$$

where τ is any integer. Every value of τ defines a possible stationary state of the atom in which it is supposed not to radiate or absorb energy,—which it can only do while passing between two states, radiating when coming nearer to its nucleus. The energy of the atom is easily found to be, in so far as it depends on τ,

$$W = \frac{2\pi^2 me^4}{h^2\tau^2},$$

and the radius is proportional to τ^2. The radius of the atom is of course the radius of the orbit. When the atom passes from a state defined by τ_1 to one defined by τ_2, the energy emitted, which is equated to $h\nu$, and shows frequency ν, is

$$h\nu = \frac{2\pi^2 me^4}{h^2}\left(\frac{1}{\tau_1{}^2} - \frac{1}{\tau_2{}^2}\right).$$

This is Balmer's formula, in essentials, but more surprising, the numerical value of

$$\frac{2\pi^2 me^4}{h^3}$$

is the actual spectral constant of Rydberg, at least to two or even three significant figures, which are all we can test.

This theory was even more strikingly successful in its application to the charged atom of Helium,—nucleus $2e$, but only one electron. It gave an account of some historic spectral series which had been supposed to be due also to Hydrogen, but which, as later experimental work has confirmed, actually must be accredited to helium.

Another point in which this theory was significant from the outset is that it gives spectral frequencies as differences of two functions of integers, as the well-known Combination-Principle of Ritz demands.

But much more work has ensued recently. The simple circular orbit of the electron discussed by Bohr is highly unlikely to be the usual one. We are all aware, even in Particle Dynamics, of the very restricted conditions of projection of a particle, under the action of a central force, which allow it to

describe a circle. Under the law of inverse square of distance, it generally describes an ellipse, which involves non-uniform velocity. In order to deal with the possible paths of a non-circular type, we must take account of the fact that the mass of the electron varies from one point of its orbit to another. As only certain circular orbits are possible, we expect also that only certain elliptic or more general orbits will be possible also. What is the more general form of the second restrictive principle for such orbits ?

The answer to the question was suggested by Prof. W. Wilson, and very soon afterwards, independently by Sommerfeld. In order to explain it, we must turn for a moment to general Dynamics as formulated by Lagrange. If a system has n degrees of freedom, it can be defined, as regards all its points, by n quantities $q_1, q_2, \dots q_n$, which may be lengths, angles, direction cosines, and so forth. If its kinetic energy is T, we call $\partial T/\partial q_r$ the generalised momentum corresponding to the coordinate q_r.

Let these momenta be $p_1, p_2, \dots p_r$. Then the summation

$$p_1 dq_1 + p_2 dq_2 + \dots + p_n dq_n$$

has great importance in general dynamical theory, as also has the integral derived from it. The generalisation which has proved successful in extending the scope of the Quantum theory is to the effect that for each coordinate and its corresponding momentum, we have a relation

$$\int p_r \, dq_r = n_r h,$$

where n_r is an integer. The motion is regarded as pseudo-periodic, or in other words, thoûgh the whole system may not have a periodic motion strictly so-called, each coordinate repeats itself after a cycle of values, and the integral is to be taken throughout such a cycle. Of course, physics has no interpretation or explanation to offer for such a curious relation. We can only say that it works, or does interpret the phenomena.

We may notice that every coordinate introduces a quantum integer n. In two-dimensional elliptic motion about a nucleus, the possible ellipses, if we use polar coordinates, are restricted by the relations

$$\int m\dot{r} \, dr = n_1 h, \qquad \int mr\dot{\theta} \, d\theta = n_2 h$$

—when we insert the momenta appropriate for such coordinates. This is on the supposition that we neglect the change of mass of the moving electron with its velocity. The curious result occurs, that if these conditions replace the two " initial " conditions which would be used in ordinary dynamics, the total energy of the elliptic orbit to which the integers n_1 and n_2 belong, becomes

$$W = \frac{2\pi^2 me^4}{h^2} \cdot \frac{1}{(n_1 + n_2)^2}$$

—a result which cannot be foreseen at any stage of the analysis, and which gives a shock at first sight. For $n_1 + n_2$, of course, behaves like a single integer, with the astonishing result that even when the electron is allowed to describe its possible elliptic orbits, the spectrum of the atom is just the same as though they were the simple circular orbits. That this is no accident, but a real piece of evidence that we are on the track of a real interpretation of the invariability of emission spectra of atoms, could hardly be disputed even at this point.

Fortunately, we can go much further. When the variation of mass with velocity is taken into account, we find that each possible ellipse above becomes a group of ellipses nearly coincident, with values of W, the total energy, only differing slightly, so that the possible stationary states occur in groups as do, in consequence, the spectral lines emitted during the passage between two of these states. Thus we obtain the " fine structure " of the lines in the Hydrogen spectrum, each " line " being an assemblage of several, very close together and

almost overlapping. These separate lines of a group are called "components," and their distances apart, on the wave-length scale, the "separations."

The groupings given by this theory have been verified in the laboratory, and even more striking is the verification, by Paschen, of the similar fine-structure found in the spectrum of charged Helium,—more readily measured because the separations are greater.

Time does not permit an account of the further verifications,—the manner, for instance, in which theory predicts the splitting of the lines under the action of an electric field, and gives the separations quantitatively with great accuracy. There can be little doubt that, in the main, the theory is on correct mathematical lines, difficult though it be to understand the why and wherefore of the curious restrictions which the Quantum theory imposes on electronic motions in the atom. There is something very peculiar about a "bound" electron, or one forming a definite part of an atom, which distinguishes it entirely from an electron, however near to an atom, which is not so "bound." Many physicists contend that tubes of force are a real structure in the aether, and that a bound electron is attached to the nucleus by some such tube which has a real concrete existence and is not merely a convenient mathematical fiction.

It certainly seems, on experimental grounds also, that the difference between a bound and a free electron is vital. For there is even a body of evidence tending to show that an electron bound in an atom loses its repulsion for external electrons. For example, Prof. Merton and myself examined the spectrum of Hydrogen as shown in a tube full of Helium at a pressure of 42 mm. of mercury, and the fourteenth member of the Balmer series could be seen plainly, though calculation shows that, under these conditions, the atom emitting this line was twice as large as the average distance between the atoms. Yet it emitted the proper radiation, quite undisturbed by the other atoms.

Perhaps I have succeeded in showing the nature of some of the problems now presented, which call urgently for mathematical suggestion and help. The theory is in its infancy, but there can be no doubt that it has superseded, finally, all previous theories of atomic structure. J. W. NICHOLSON.

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104. It is almost impossible to discredit Woodhouse's remark: "Whether I have found a logic, by the rules of which operations with imaginary quantities are conducted, is not now the question: but surely this is evident, that since they lead to right conclusions, they *must have a logic*."—De Morgan's *Trig. and Double Alg.*, p. 41 (footnote), 1849.

105. That ever you should pitch upon me for a mechanic or geometric commission! . . . I know no more of a Latin term in mathematics than Mrs. Goldworthy had an idea of verbs. I will tell you an early anecdote in my own life and you shall judge. When I first went to Cambridge I was to learn mathematics of the famous blind professor Sanderson. I had not frequented him a fortnight before he said to me, "Young man, it is cheating you to take your money: believe me, you can never learn these things; you have no capacity for them." I can smile now, but I cried then with mortification. The next step, in order to comfort myself, was not to believe him: I could not conceive that I had not talents for anything in the world. I took, at my own expense, a private instructor (Dr. Trevigar), who came to me once a day for a year. Nay, I took infinite pains, but had so little capacity, and so little attention (as I have always had to anything that did not immediately strike my inclinations), that after mastering any proposition, when the man came next day, it was as new to me as if I had never heard of it; in short, even to common figures, I am the dullest dunce alive. . . .—Walpole to Sir Horace Mann, Dec. 13, 1759.

VECTORS.

BY PROFESSOR C. GODFREY, M.V.O., M.A.

VECTORS in two dimensions are generally studied in schools; I do not suppose that the study is often pushed into three dimensions. But it is difficult to understand a modern book on electricity without some knowledge of three-dimensional vector notation.

The particular point that I wish to bring before you is that the two-dimensional case is often presented in such a way that its relation to the more general case is obscured.

The particular kind of spatial vectors that I shall consider are Hamilton's (see *Encycl. Brit.*, art. 'Quaternions,' by P. G. Tait). According to Hamilton, the product of two vectors α, β, of lengths a and b inclined at an angle θ, is the sum of two parts, a scalar part and a vector part. The scalar part, $S\alpha\beta$, is $-ab\cos\theta$; the vector part, $V\alpha\beta$, is perpendicular to the plane of $\alpha\beta$, and is of length $ab\sin\theta$.

Now in the elementary two-dimensional presentation it is often said, loosely, that the product of two vectors in a plane is a third vector in that plane, obeying the De Moivre law $(r_1, \theta_1)\times(r_2, \theta_2)=(r_1r_2, \theta_1+\theta_2)$. This is confusing; it yields a product agreeing neither with the scalar product nor the vector product of Hamilton. The De Moivre rule in fact applies to the multiplication, not of vectors, but of complex numbers; and it is only fair to say that many text-books make this clear. A complex number is the ratio of two vectors. The matter becomes quite simple if a unit vector, say σ, along OX is introduced; then the general vector in the plane is $r(\cos\theta+\sqrt{-1}\sin\theta).\sigma$, so that $r(\cos\theta+\sqrt{-1}\sin\theta)$ or $re^{\sqrt{-1}\theta}$ is not a vector but an operator; and the unit vector along OY is not $\sqrt{-1}$ but $\sqrt{-1}.\sigma$. De Moivre's rule then applies to the multiplication of two operators and not of two vectors.

In elementary work we never multiply one vector $r_1e^{\sqrt{-1}\theta_1}.\sigma$ by another vector $r_2e^{\sqrt{-1}\theta_2}.\sigma$.

Now the loose identification of vector and operator is harmless in two dimensions; the unit vector σ occurring linearly in all the equations 'cancels out,' and is forgotten. But it has to be resurrected if we are to understand the two-dimensional case as a particular case of the general.

In the general theory, the ratio of two unit vectors, *i.e.* the operator needed to convert the 'denominator' into the 'numerator,' is $\cos\theta+\omega\sin\theta$, θ being the angle between the vectors and ω the unit vector perpendicular to their plane. This is a quaternion. It means 'Turn through θ about the axis ω, the vector operated on being perpendicular to ω.' And

$$(\cos\theta+\omega\sin\theta)(\cos\phi+\omega\sin\phi)=\cos(\theta+\phi)+\omega\sin(\theta+\phi),$$

so that $\cos\theta+\omega\sin\theta$ may be identified eventually with $e^{\omega\theta}$.

Unit vectors along the three axes are written i, j, k; and $i^2=j^2=k^2=-1$, $jk=-kj=i$, etc.

Returning to the elementary two-dimensional theory in the xy-plane, ω becomes k, while our σ is i (for the present we must distinguish between i and $\sqrt{-1}$). Thus the geometrical plane interpretation of complex numbers is from this point of view concerned with the form $r(\cos\theta+k\sin\theta)$, which must be regarded as a quaternion. The general vector in the xy-plane is

$$r(\cos\theta+k\sin\theta).i=r\cos\theta.i+r\sin\theta.j.$$

But we are not troubled by the presence of i and j, for the i cancels out of our calculations, and leaves only k of the trio.

Now it may be objected that we are not allowed to identify $\sqrt{-1}$ (in $e^{\sqrt{-1}\theta}$) with k; for multiplication of complex numbers is commutative, whereas i, j, k multiplication is non-commutative. But i, j, k are commutative with scalars,

so that when we are concerned with nothing but k and scalars, the non-commutative character of k is not developed. And as $k^2=-1$, there is no reason why the flatlander should not identify k with $\sqrt{-1}$. And if he has no occasion to retain i (the unit vector along OX) in his calculations, he needs not consecrate the symbol i to this use, and may replace k by i; which he does.

If we may envisage the elementary theory as a particular case of the general theory as above, an obstacle to progress is removed; many students must have been troubled by the apparent want of harmony between the two.

The only objection that I can see to this proceeding is concerned with biquaternions. The general quaternion is $a+bi+cj+dk$, where a, b, c, d are real, a quadruply-infinite aggregate. If each of the a, b, c, d is complex, we have Hamilton's biquaternion; thus

$$a+\sqrt{-1}\,a'+(b+\sqrt{-1}\,b')i+(c+\sqrt{-1}c')j+(d+\sqrt{-1}\,d')k,$$

an octuply-infinite aggregate. These biquaternions are used in electrical theory and optics. Tait explains that the $\sqrt{-1}$ occurring here is a scalar, and the situation becomes for the moment rather baffling; but Tait does not vouchsafe any further help than the remark that "some little care is requisite in the management of these expressions." If we descend again to Flatland, we have

$$a+\sqrt{-1}\,a'+(b+\sqrt{-1}\,b')k,$$

which has ∞^4 values; and though $\sqrt{-1}$ is commutative with k, and $k^2=-1$, we cannot identify k with $\sqrt{-1}$ without degrading the quadruply-infinite into the doubly-infinite.

But there is no need to reduce the degrees of freedom in Flatland to two unless we are concerned solely with the representation of points in the plane; and this limitation would equally exclude biquaternions in space.

The bivector $\qquad \omega=(a+\sqrt{-1}\,a')i+(b+\sqrt{-1}\,b')j,$

or rather its 'real' (non $\sqrt{-1}$) part, would be appropriate for the representation of motion in an elliptic orbit about a centre of force attracting as the distance. For we have only to put

$$a+\sqrt{-1}\,a'=pe^{\sqrt{-1}\,\alpha}, \qquad b+\sqrt{-1}\,b'=qe^{\sqrt{-1}\,\beta},$$

and then the real part of $\omega e^{\sqrt{-1}\,\mu t}$ is seen to give the elliptic orbit with its five specifications (*e.g.* axes, inclination of axes, epoch, and period).

To resume, the complex number may be interpreted geometrically as a Hamilton quaternion in Flatland; and this aspect is brought out by insisting on the necessity of the (latent) unit vector along OX.

I have not argued that for practical purposes the Hamiltonian system is to be preferred to other more modern systems of vector analysis.

C. GODFREY.

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106. Old John Orr, as already said, used to come much about Brownknowe; being habitually *itinerant*, and (though Schoolmaster of Hoddam) without settled home. He commonly, my Father said, slept with some of the Boys, in a place where (as usual) there were several beds. He would call out from the bed to my Grandfather, also in his: "Gudeman, I have found it,"—found the solution of some problem or other, perhaps arithmetical, which they had been struggling with. . . . He was my father's sole Teacher in "Schooling."—*Reminiscences*, by T. Carlyle, edited by C. E. Norton, p. 32.

107. A profound algebraist once mentioned to the writer (Charles Butler, Esq., of Lincoln's Inn, *Reminiscences*, 1824) that he had never found in that science, a problem, the solution of which required greater mental exertion than passages occurring in almost every page of this celebrated commentary. —[Coke upon Littleton. Who was it? Pollock, Alderson, Maule, Jacob, . . . ?]

THE DALTON PLAN AND THE TEACHING OF MATHEMATICS.

By Miss F. A. Yeldham, B.Sc.

The Dalton Plan, which has been discussed so much recently, is a change of organisation within school which affects a girl's whole school life and her general development more than her work in individual subjects. I am obliged in describing it to devote a few minutes to speaking of general school life, and when I turn to the teaching of mathematics there seems so little to say, that could not be said as appropriately of teaching in ordinary circumstances that I do not know if it will justify the consideration of this meeting.

There has been a certain organisation in schools since girls' secondary schools opened, and the women from them who have had the enterprise to give public service in so many and new directions are evidence of its success in the main. There were a few points in this organisation which appeared as defects, and the aim of the Dalton Plan is to remedy these.

The chief failing perhaps was due to the desire on the part of teachers to have a well-ordered school with every movement well thought out and arranged beforehand for the benefit of the girls, and the forgetfulness that the autocracy which succeeded best in this, did so at the expense of the full and free development of the individual. In the extreme case we arranged every minute of a girl's life for her from ten minutes before prayers in the morning, until she was out of the school gates in the afternoon. Whatever the importance of the subject under attention, at each bell, several in the course of the day, she must stop doing it and do something else. There was a consensus of opinion that a girl's interest in any subject lasted exactly forty minutes, and that the periods of forty minutes must be synchronised for the few hundred girls in the school. Each lesson, therefore, was brought to a close with the bell, and then we did something else, not with natural zest and interest so much as from a sense of loyalty to the time-tables which had been approved by the Board of Education.

Most of the intervals between the bells were used for lessons. For several years Inspectors have inspected teachers, not scholars, and the teacher's part has been the active part in the class room. Teachers "gave lessons," and many have interpreted this to be freedom of movement, freedom of speech and freedom of thought for themselves, while the girls sat receptive, quietly in their seats raising a hand as a prelude to speech and bending forward only upon permission to write. A teacher could lead a lesson where she would. Sometimes a girl would ask a question, but to attract attention to herself by asking a series of questions and insisting on a point until she understood it, while she was a member of a class, required real courage, and to explore the bypaths into which her individual whim took her when the teacher preferred the straight road, showed a forwardness which was not quite right form.

Even at home the girl was controlled from school. Preparation had to be done on certain evenings in a definite time, and there was quite as much censure for overtime spent upon it as for undertime.

The scheme of work for the year was known to all in authority, but the girl rarely saw more of it at a time than the lesson being given, and the preparation set upon that lesson.

The Streatham Secondary School has over 780 pupils. The Upper School, of about 530, adopted the Dalton Plan sixteen months ago, and the Junior Departments have had a modification of it in operation since last September. It is still in the experimental stage, and we are still making modifications.

By the Dalton Plan as we know it, the *assignment*, or programme of work to be done in the course of the month, is given to the girls. It tells them the matter to be taken and should give guidance as to the method of attack.

The number of lessons is kept down to a minimum, in Mathematics it is three periods of forty minutes a week in the Second and Third Forms, and two periods a week in the Fourth Forms and above. The subject mistress has one other period when she may call together a group of girls (not a class) to explain a point, and for the remainder of the time the girl does the work when she chooses. At school, as far as possible, she works in the subject room, where she may consult the mistress or a book in the subject library. The mistress uses her lesson time as she thinks best, for lesson, or discussion, or free study. At tutorial times she is consulted, or she corrects work or tests individual girls in any way she finds suitable.

The girl must satisfy the mistress on one assignment before she begins the next. Because of varying abilities the assignments are now graded. The *lower grade* is within the range of the slowest girl and must be done by all, the *middle grade* gives opportunity for wider reading and deeper thought, and the *higher grade* encourages the brilliant girl to study as far as she can, but neither middle nor higher grade may encroach upon the next assignment.

The approximate apportionment of periods (that is forty-minute periods) per week is posted in the rooms, and it is each girl's responsibility to see that she arranges her time with sense and forethought, keeping a balance between procrastination and overwork and worry, while she finishes her work to time.

The girl holds a card throughout the month. On one side she keeps account day by day of the quality of work done, and on the other side as each assignment is finished, the subject mistress enters her remarks upon the quality of it. These cards are much less formal than the reports sent home at the end of term and much more intimate between girl, mistress and parent. They are used for criticism and advice, and both are given quite frankly. There is also a chart in each subject room for all the class. This is the property of the mistress, but the entries on it are made by the girls.

It is hoped that giving a girl more responsibility and practice in using judgment in distributing her time wisely among all her conflicting claims will be a better preparation for her for life. Another generation will be able to say if this is so. In a matter more immediately shown, it is hoped that when turning to a subject at times of her own choosing, she will come to it fresher and with more enthusiasm, and will take more interest in her own education.

Before saying anything of this year's experience or making any criticism, I should mention that my subjects are Mathematics and Physics, and that I am not responsible for either the Mathematics or the Science of the school. In any new venture each one sees a different aspect, and I think you would prefer that I give quite simply one or two points which have occurred to me during the change.

The first point I noticed eighteen months ago, when the Plan was put to us, was that the checking up on girls' and mistresses' charts emphasised quantity of work done. Perhaps nowhere is this less defensible than in the subject of Mathematics. A mathematics assignment on its lowest plane consists of a number of exercises to be worked and shown up. Checking off on the charts a number of exercises worked through eventually correctly or showing a file full of neatly copied work may satisfy a mistress as quantity of work done, but it is no guarantee of a principle understood or a power gained. It is a confusion of idea that quality of work, or, better, mastery of work is measurable as quantity written.

If a girl is given an assignment which is merely a number of questions to work and allowed to get any help she pleases, she may take the order literally and feel the assignment properly carried through when the answers are written, whether she understands what she has written or not. I know from the remarks dropped by others that the view still obtains that mathematics is merely written work which can be cleared off quickly. In class teaching there were short tests, when girls worked unaided and to time, which showed the

teacher where help to understanding was needed, and something to serve this purpose is still necessary. It is almost a tradition to give girls sums and exercises only, leaving them to become more skilful as they pass up the school in divining the purpose of the work given them, but if a girl is to be responsible more than before for educating herself she must be let into the secrets of the aims of her task. I do not see why an assignment could not say " Here is a principle to understand and it will help you to apply it to these questions, or " You need to be quick and accurate in manipulating these symbols, practise on these exercises." We are never sure exactly how much children read between the lines, and in some way or other we should keep before them the view of exercises as stepping-stones only to the bank of understanding and power. Robert Recorde spoke like this to his scholar in 1543, and did no harm by taking him into his confidence about his own mathematical education. The checking up then would be " I understand," or " I am skilful," rather than " These answers are right."

In the grading, if the aim of the work is *to gain a power, e.g.* facility in the manipulation of symbols, some girls need more exercises for practice than others; what is barely sufficient for the slow, probably involves a marking of time for the quick—only a little on each rule, but during school life a considerable wastage. The Dalton Plan should eliminate unproductive labour. If the quick girl may stop working the lower set of exercises on the rule as soon as she has gained power and find in the higher grade exercises combining this rule with another, so as to introduce a little difficulty, the aim of the grading seems to be satisfied. Her time and effort are being better used than in preparing paper for the waste paper basket by completing the lower assignment.

On a short assignment it is more difficult to grade when there is *a principle to be understood*, because in understanding a principle a girl either understands or she does not understand. I will illustrate from Applied Mathematics. If the assignment asks for the principle of the lever there is no half-understanding about the sum of the moments of the forces. If the principle is understood the girl can see balance, crowbar, nutcrackers, etc., as easy cases of levers, and can deal with the numerical values given to the several forces and their distances from the fulcrum. There is the one principle standing as a whole behind all this work, and either it is mastered or it is not mastered,—the work must be put in its entirety in the lower grade. To put in the middle and higher grades more problems on the same lines is merely amassing work, and it may become running the gamut of the multiplication table or decimal fractions for the girls, who least require the practice in them. It is not encouraging deeper thought so much as marking time. Perhaps the examination of the weighing machine at a goods station to see it as a lever and find the points of application of the forces, or looking into the wheel and axle, more nearly approximates to the idea of the grading. But it is then unfortunate, if you have asked the girls in the lower grade to look around for examples of levers in daily use, or are reserving the wheel and axle for the next lower assignment. In the higher grades of a longer assignment could come the questions which call for selection of knowledge from a wider field and its orderly application to the data.

To describe briefly the work of one month, I am taking an assignment of the Fourth Form Applied Mathematics. The assignments are given out. We can rely on four long lessons only, but the laboratory is open most of the week, and girls come in as they please unless, as sometimes happens, we are crowded and have to ask a few to go elsewhere and write up notes. The assignment began with four questions on the windlass and capstan, seeking all we thought a girl of fourteen could be expected to master of them. An unemployed ex-soldier made us a model of a windlass, which we set up over a sink and which works splendidly, even to showing friction. Everything is done practically first and the mathematical argument follows. Working alone

the girls took first the question on the meaning of mechancial advantage. They lifted the 200 grs. weight directly and by turning the handle, and the advantage of the latter method was very noticeable. A spring balance put to the handle registered about 45 grs. Then they examined and described the machine. In the lesson we concentrated on the drum and arm, and found something akin to a lever in it, an idea which became more apparent when the machine was looked at from the end and drawn in plan. Then it showed that the mechanical advantage was dependent on the ratio of the radii of arm and drum, and these were measured and the ratio found. Lastly, reasons had to be found for the mechanical advantage found in the two ways not being in exact agreement. One question on pulleys asked about the single pulleys in map supports and windows. It brought on a discussion, repeated several times with individual girls, of window fittings generally,—sash windows and venetian blinds. The next piece of work was to set up pulleys in the two ways indicated and find practically the mechanical advantage, then to argue from previous ideas what the mechancical advantage in each case should be. The case of the weightless pulleys follows upon this as a corollary, though I suppose in theoretical work it still is taken first. There were questions on purposes to which pulleys could be put. The second lesson was used for individual practical work and discussions, because the girls by that time were at various stages. A lesson had to be given on Centre of Gravity, and the practical work on it was to find the centre of gravity of several lamina, and chair and basket. The usual explanations of overturning vehicles were asked. In the whole month it was found that only two ordinary lessons were given.

We ask for certain pieces of work to be written up carefully, others may be written or not as the girls please. I much prefer a careful and full spoken explanation, such as one is so often called upon to give in life, to a scanty written one. No guidance is given in writing notes unless it is seen to be required. We assume that if our points are clear and in order in lessons and discussions, they will remain clear and in order in the writing up. Written work is corrected at any time. The wise girl offers it frequently and keeps her book right. It is, however, very easy to say that to-morrow will do, and a mistress who receives a whole month's work to correct on the last day of the month generally sees to it that it does not happen a second time.

Some find that correcting is not so monotonous now that one or two books are offered at a time, but in Geometry or in any case where there is an argument to follow closely, there seems an advantage in correcting several, say a dozen or eighteen, while the mind is on it.

The girls have responded well to the scheme, and, with a few exceptions, they are taking their work earnestly and sensibly. We have always trusted our girls, and so have had their co-operation from the beginning. The majority choose their own grade—lower, middle or higher—satisfactorily, but there are always a few whose confidence is greater than their power, and a few who will do the minimum. They are finding corrections more helpful when they stand by the mistress and there is immediate understanding and overcoming of the difficulty. Since there has been tutorial work and easier access to the mistress, the diffidence about asking questions during lessons has vanished. In fact, the tables may be turned and the girls take the active part in a lesson, and by their insistent questions spur a mistress on to give more than she intended. This happened in the lesson on centre of gravity. The class led the way, and I now understand that the lesson should be the starting point of enough threads of thought to last a whole term—in fact, we have had to send the overflow up to the Science Club for discussion. It has been said that if a lesson is properly given, there should be no need to ask any questions on it. I may be wrong, but I prefer the lesson which leaves a girl with the thought that there is more beyond which is worth her enquiry. A few points begun

in this lesson were: the laws of motion, the path of a projectile influenced by several forces, resolution of forces, the idea of slices of infinite thinness of objects, which they would like to apply another way to the common geometrical figures, triangle and parallelogram, thinking of them as solids of infinite thinness rather than as surfaces of two dimensions only, and thus getting new and practical properties to medians of triangles and diagonals of parallelograms. This can become diffuse, and one is obliged to ask for precision of thought and statement at times, but these girls can and do sit down to their books afterwards and struggle to get the word or phrase to give the right meaning to their notes. The freedom of the Dalton Plan has been a great help in awakening interest, but I attach more importance to the conviction of the girls that Applied Mathematics holds something of practical value in life.

There is one great difficulty in Mathematics which I have left to the last—books. Subjects like English, History and Geography have in the last twenty years added many well-written books suitable for use in schools either as text-books or library books. In Mathematics most, not all, of the histories are written for older students. There are not many lives of mathematicians written simply and interestingly—this point was put to me a few months ago by the Oxford Press. Mathematics has served the needs of men from time immemorial, and is behind every activity of his, and the story of old-time Mathematics could be made most interesting, if any mathematician had the gift. When our girls are in English, History and Geography rooms they may if they wish take a book and read on the boundary of, if not a little beyond, the assignment, and the time is not ill-spent because of the interest it awakens. They are just as interested in books on Mathematics. A few of my books were in my room last year and they were constantly borrowed, and brought up to me to be discussed. But the pity is that there are not enough suitable ones.

In text-books, Histories and Geographies have long since given up being lists of dates and events, rivers and towns and exports. Their writers have realised the value of description and that the development of a country is a living story. Mathematics books are still very cut and dried. Books in other sciences suggest experiments which lead to the discovery of truths—many mathematics books still state only the results of experiments. In Mathematics the majority of books are unsuitable for the Dalton Plan, being written during the years when classes depended upon teachers for good and frequent lessons. There are teachers' editions written in too advanced a style for girls, and smaller books of sets of examples for practice.

With lessons cut down to a minimum, good explanatory books would be a help, but they must be written by some one who knows the subject he is writing and its simple applications. It should not be necessary to say this, but I have recently noticed a few points in books which are being recommended for school children—a section devoted to practice in substituting abstract numbers in physical and other formulae to the wholesale confusion of units and dimensions, exercises to which arbitrary answers have to be given with now and then a question contradicting the one immediately before it, a section stating formulae of all sorts and kinds, sometimes giving a meaning to the symbols, but never any indication as to the building up of the formula, and sums to be worked on these, much reproduction of type sums without explanation, merely unintelligent rote work, and inaccurate mention of facts from all the sciences and industries of man. They may be very interesting and suggestive books, but they are most harmful in the hands of an unqualified teacher. From the same source comes their own indictment,—"It is a well-known fact that children do not apply their Mathematics in the Physics room."

I am dwelling rather long on this point, but as auto-education, and with it self-expression, extend we shall find that what we offer children will come

under review, and I should like to see improvements in readiness. Perhaps I may mention the lines on which I think one or two of the children's demands will come.

1. Much of the introductory Mathematics can be learnt in a laboratory as any other science. Children enjoy handling matter, and Mathematics was Egyptian before it was Greek. A book could suggest how to investigate: space and its measurement in volume; surfaces, developable and undevelopable, their formation and characteristics and area; mass and weight. There is a paragraph or so on part of this in *A Little Book on Map Projections*, by Mary Adams. It is simple and explanatory, and I do not see why the whole of this section cannot be written up in a similar style in a book to be used for revision.

2. The chapter headings of school mathematics books should be reconsidered. I do not mean that many would be rejected, but that we should have asked ourselves again in these altered circumstances of education upon what grounds each can claim its place.

I hope, in spite of my criticisms, that I have left the impression that the Dalton Plan is a possible organisation in a school, and probably much preferable to the old. F. A. YELDHAM.

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108. Rigby Kewley, Rector of Baldock, Hon. Can. Rochester, tells of Kingsley's Tripos, 1842: One question remained. . . . Describe a Common Pump. Of the internal machinery of the pump, Kingsley was unable to render a scientific account, but of the outside his vivid imagination supplied a picture which his facile pencil soon transferred to paper. Under the heading 'Describe a Pump,' he drew a grand village pump in the midst of broad grins, and opposite the porch of an ancient church. By the side of ye pump stood in all pomposity of office, ye village beadle, with uniform and baton. Around were women and children of all ages, shapes, dress, and sizes, each carrying a crock, a jug, a bucket, or some vessel large or small. . . . Around the pump itself was a huge chain, padlocked, and surrounded by a notice:

This pump locked during Divine Service.

This Kingsley sent up to the examiners as his answer to the question. . . . So clever that the moderator of the year had it framed and hung up on the wall of his room.—*Life and Letters of Kingsley*, i. 62. [Frost's *Collection* covering this date omits the Bookwork of 1842 and other years, so I cannot check the *fact* of the *question.*]

109. Jane Welsh Carlyle. School at Haddington: Boys and girls went to the same school; they were in separate rooms except for Arithmetic and Algebra. Jeannie was the best of the girls at Algebra. . . . She was always anxious to work hard, and would sit up half the night over her lessons. One day she had been greatly perplexed by a problem in Euclid; she *could not* solve it. At last she went to bed; and in a dream got up and did it, and went to bed again. In the morning she had no consciousness of her dream; but on looking at her slate, there was the problem solved.—Carlyle's *Reminiscences*, p. 54.

110. Brougham is all day long working sums in algebra, or extracting cube roots, and while he pretends to be poring over the great book (the cases of the parties) before him, he is really absorbed in his own calculations.—*Greville. Memoirs*, Dec. 2, 1838.

111. Alfieri, at the Academy of Turin, "learned a little Latin, and tried, in vain, to acquire the elements of mathematics; for, after the painful application of several months, he was never able to comprehend the 4th proposition of Euclid, and found all his life after that "he had an anti-geometrical head."

THAT PURE AND APPLIED MATHEMATICS OUGHT TO BE TAUGHT AND DEVELOPED *PARI PASSU* IN BOYS' SECONDARY SCHOOLS.

By A. Dakin, M.A., B.Sc.

It is now a commonplace of History, that Pure Mathematics had its origin in the practical needs of man, and that at intervals it has renewed its vitality by turning to its practical application. It has, during recent years, become increasingly recognised that the bases of mathematical education are the concrete and the practical; and this Association, by the Reports and Papers published in the *Gazette*, has emphasised the view that "the utilitarian aspect and application of Mathematics should receive a due share of attention in the mathematical course." This increasing respect for the concrete and practical has not only led to considerable changes in the teaching of Arithmetic, Geometry and Algebra, but has brought into the Schools closer co-operation between the teachers of Mathematics and Science, and has resulted in a very desirable correlation of the syllabuses in these subjects.

We have in one branch of Applied Mathematics, viz. Mechanics, a subject which, though based on experiment, is particularly well adapted to Mathematical treatment, and which should prove especially valuable as a link between the Mathematics and Science Departments of any school. The object of this paper is to suggest that alongside of the Pure Mathematics Course in every Boys' Secondary School there should be a Course in Mechanics starting at about the age of twelve or thirteen, and continuing throughout the school life. This is no new idea. In fact, when I received the invitation from the Council to open this discussion, my first impression was that the topic was ten or fifteen years out-of-date. Ten or twelve years ago the study of Mechanics was commenced in the schools at the age of sixteen years or after, and many boys left school without any knowledge of the subject beyond that obtained in a short and totally inadequate course taken in the Physical Laboratory in the III. or IV. Forms. It is, however, within the experience of the Members of this Association, that there has been a great improvement in this respect during the last ten or fifteen years. The Report on the Teaching of Mechanics issued by this Association in 1918 recommended that the teaching of this subject should be begun early. About thirteen was the age suggested; and, attached to that report is an Appendix giving four alternative schemes in Elementary Mechanics, arranged by well-known and experienced teachers for pupils between the ages of twelve and sixteen. Some of these schemes, at first sight, appear to be based upon an unduly optimistic estimate of the genius of the teachers and of the capacity of the pupil, but experience proves, that with the average teachers and the average pupil, a fair proportion of the work outlined in the most ambitious of the four schemes can be profitably taken before the age of sixteen.

Again, in December, 1919, the Committee of this Association published a "Report on the Teaching of Mathematics in Public and Secondary Schools," in which occur the following Recommendations:

"That the Mathematical course in the earlier stages should not be concerned exclusively with Arithmetic, Algebra and Geometry; that such subjects as Trigonometry, Mechanics and the Calculus should be begun sooner than is now customary, and developed through the greater part of the boy's school career."

"That no boy should leave school entirely ignorant of Applied Mathematics (*e.g.* Mathematics relating to the Stability of Structures, Motion of Bodies, Electrical Plant, Astronomy, etc.)."

The report further emphasises the value of "lapse of time," in dealing with

any subject in order that the ideas peculiar to that subject may be thoroughly assimilated.

It would thus seem that in opening this discussion, I am whipping a willing horse, but enquiry shows that there are still Secondary Schools giving little or no instruction in Applied Mathematics, and even now, as Mr. Fawdry told us two years ago, many boys leave school without any knowledge of the subject at all. Examination statistics show that in the various First Examinations (School Certificate and Matriculation Examinations), the ratio of the total number of candidates taking Pure Mathematics to the number taking Mechanics is very nearly 8 to 1. This means that in many schools, the subject is either neglected altogether, or postponed until the age of sixteen, when the pupils have taken their first examination. For, owing to the present overcrowded state of the curriculum of Secondary Schools, any subject not required for the first examination, is bound to suffer some measure of neglect in the years when pupils are preparing for that examination.

This neglect in the past has been largely due to the difficulty of the subject. The subject of Mechanics, based as it can be on surprisingly few principles, is admittedly difficult both to learn and to teach.

Even when based upon the large body of intuitional knowledge that every boy brings to its study, and upon experiment, which alone makes it a school subject suitable for boys of thirteen or fourteen, it still remains a difficult subject. But this difficulty is counteracted by the interest that every boy brings to its study, an interest which is partly inherent to the subject, and which partly arises from the fact that three out of every five boys at the age of thirteen or fourteen hope to become engineers or draughtsmen. Every boy has an interest in things mechanical, and unless you draw out from his intuitions and experience correct and accurate mechanical principles, he will evolve for himself principles which are likely to be erroneous and inaccurate.

Every teacher of Dynamics in a town must at times have marvelled at the detailed knowledge of motor-cars and motor-bicycles possessed by the average boy. A certain and speedy way to improve the teaching of Dynamics in the schools would be for the State or the Local Education Authorities to present every teacher of the subject with a motor-car! However, we must look for less direct but more probable remedies, and the one method open to us is for this Association to press, both in season and out of season, for more attention to the subject in the schools.

The subject will be developed under different circumstances in different ways, and it is, of course, essential that liberty should be left to each school to develop it on its own lines, but it may possibly give "reality" to this discussion if I briefly outline a method of which I have had experience, but for which I claim no novelty. Many pupils enter the Secondary School from the Elementary School at the age of twelve, and for the purpose of this description, I will take that as the beginning of the normal school life. For the first year there is no formal treatment of Applied Mathematics, but the pupils take a course in weighing and measuring, finishing with the Principle of Archimedes, and its simpler applications. The next year, after in many cases only one year's teaching of Algebra and Geometry, a course in Mechanics is begun, consisting mainly of experimental work and numerical examples on Springs (both in extension and compression), Levers, Moments, Parallel Forces, Simple C. of G.'s, Work, Machines, and finally the Triangle and parallelogram of forces. The fact that this preliminary course ends instead of begins with the parallelogram of forces, is, of course, not novel, but it is worth noting because, in my opinion, the fact that Statics is no longer based on the parallelogram law makes the introduction of the subject possible at an earlier age than hitherto.

During this experimental course, "talks" are given at intervals on the principles involved and also occasionally on the history of the development

of the subject. During this year proportion and easy numerical trigonometry are taught simultaneously with the statics, and continual cross-references are made between the Pure and Applied Mathematics.

The following year, Dynamics is treated in a somewhat similar manner, except that in this subject more appeal is made to intuition and fewer opportunities arise for experimental work. (Fletcher's trolley, of course, supplies most of the experiments at this stage.) The main aim is to develop the principle that variation of motion, both *in* straight-line motion and *from* straight-line motion must be explained in terms of "force." After a general talk intended in the main to guard against the possible misconception that rectilinear motion is the only possible one, the pupils discuss such motion, obtaining both acceleration and distance from a speed-time graph; they then pass to the case of uniformly accelerated motion, to motion under gravity, and then to the relation $\frac{P}{W}=\frac{f}{g}$, and finish with some easy examples of the Principles of Energy and Momentum. During the year occasional talks are given on Astronomical subjects (*e.g.* on the motion of the planets and of comets), and these help to build up the idea that variation *from* straight-line motion must also be explained.

During the fourth year of the school course, both Statics and Dynamics are taken; topics omitted from the preliminary course are discussed and the whole subject consolidated. At the end of this fourth year, the pupil normally takes his first examination, and then either leaves or passes on to an advanced course. Pupils, proceeding to an advanced course in Mathematics and Science, can now discuss, in the first term of that course, Dynamics of Rotation.

The value of such a course both to the pupil and to the school cannot be overestimated. The pupil has his appetite for things practical and useful at any rate partially satisfied, he finds a new interest in the lives and work of Galileo and Newton; the body of his mathematical knowledge is enlarged and new life is given to the whole.

The advantages to the school are evident: "Reality" is given to all the mathematical teaching; artificial examples in Algebra and Numerical Trigonometry are replaced by natural and directly useful ones; the Dynamics teaching supplies a stock of ideas which form an invaluable introduction to the Calculus; in addition the course provides a link between the Mathematics and Science teachers of the school, which ensures true correlation and real co-operation.

There is one other point—so far, I have restricted my remarks, in accordance with the terms of the motion, to Boys' Schools. When I was invited to open this discussion it was, I think, intended to cover both Girls' and Boys' Secondary Schools. I suggested the insertion of the word "Boys" before "Secondary Schools," because I am far from convinced that Mechanics should form part of the general school course for girls. My somewhat limited experience of teaching girls leads me to the conclusion that the subject makes but little appeal to them, and that consequently its teaching does not produce the same profitable results as in the case of boys. The scheme outlined is intended to meet the needs and satisfy the requirements of the average boy, and it would be a mistake to assume that it suits the average girl.

The time is approaching when Mathematics will no longer be a compulsory subject for examinations of Matriculation standard, and when that time comes there is a danger that the subject will play only a subordinate part in the education of girls. That danger will not be diminished, but will certainly be increased, by any endeavour to force upon Girls' Schools a scheme of work drawn up and arranged in the interests of boys. It may be that the question of Applied Mathematics for girls may best be solved by a course more descriptive and less computative, more historical and less experimental than the corresponding scheme for boys.

There is, however, in every Girls' School a small minority who have a very special aptitude for Mathematics, and we ought to press for opportunities for these girls to acquire some knowledge of Mechanics during their school life. From this "small but distinguished minority" come the future teachers of the subject in Girls' Schools, and it is important that all specialists and future teachers should have some knowledge of the applications of Mathematics. University Honours Courses should always comprise a compulsory minimum course in Applied Mathematics; it ought not to be possible for a girl to take up the teaching of Mathematics, having spent months or years doing "dreadful things with dreadful conics," and yet knowing nothing of the work of Kepler, Galileo and Newton.

To sum up, the suggestion is that in all Boys' Secondary Schools courses in Mechanics should be organised to continue practically for the whole of the boy's school life, and that in Girls' Schools opportunities should be provided, by the introduction of alternative courses of study, to enable those girls whc have special aptitude to take an adequate course in Mechanics while at school.

A. Dakin.

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112. Specious. *Vieta* used *Species* when letters were used for numbers in general. 'Logistics' was used for calculation in general. This introduced *Specious.* His first tract on easy algebra was *De Logisticâ Speciosâ.* Hence the language of algebra for some time was called the *Specious notation.*

113. Wordsworth went to St. John's, Cambridge, Oct. 1787. "When at school, I, with the other six boys of the same standing, was put upon reading the first six books of Euclid, with the exception of the fifth; and also in Algebra I learnt simple and quadratic equations; and this was for me unlucky, because I had a full twelvemonths' start on the fresh men of my year."—Wordsworth's *Autobiography.*

114. "By all means do not neglect your duodecimals. I was Senior Wrangler by knowing my duodecimals."—Turner, 1767. *v.* Neale's *Senior Wranglers,* p. 8.

115. As to mathematics [for her boy], the best plan is to start with practical work—surveying, practical geometry, etc., which can always be taught by the local schoolmaster.—*Taine,* advice in letter to Madame Francis Ponsot.

116. Up from High Elms with Henry Smith. As we were crossing London Bridge, the conversation turned on the importance of encouraging the study, at Oxford, of some of the less read Classical authors. . . . So we came round to the Anthologia, and he quoted the epitaph on Zosime, one of the most touching which antiquity has bequeathed to us.—Grant Duff's *Notes from a Diary,* March 4, 1878. [H. J. S., Savilian Prof. Geom., Oxford, 1861-1883.]

117. Of Winchester in 1822. Of Arithmetic I knew nothing—and of all that arithmetic should be the first step to, *a fortiori,* still less.—T. A. Trollope.

118. "Have we not now a geometrick school,
To teach the cross-legged youth—to snip by rule?

(A taylor has lately informed the publick, in most of the newspapers, that he fits his customers by geometrick rules.) *Gent. Mag.* 1782, p. 37. Epilogue intended for *The Count of Narbonne,* written by Edward Malone, Esq.

119. I am told that Sir Isaac Newton foretold a great alteration in our climate in the year '50, and that he wished he could live to see it. Jupiter, I think, has jogged us three degrees nearer the sun.—Walpole to Mason, Feb. 25, 1750.

120. *Square* originally applied only to corners of a figure or at most to right angle corners. v. *P.C.* xxii.

DISCUSSION ON "HOW TO KEEP TEACHERS OF MATHEMATICS IN TOUCH WITH MODERN METHODS AND DEVELOPMENTS."

THE discussion was opened by the Rev. E. M. Radford, who said that every sincere teacher of Mathematics would desire to possess that freshness and inspiration which alone can make his work of real educational value. The evil of staleness frequently arose from the mechanical use of traditional and obsolete methods, and much injustice was done to Training College students and other young teachers if they were prevented from using newer and more intelligent methods. Much excellent work was being done in the newer secondary schools, which were staffed by young and enthusiastic teachers. This was also the case with many of the Public Schools, but some of them were still in a backwater. He knew of one school at least in which there had been no change in the mathematical text-books for the last twenty years. More stress should be laid on *methods* of teaching, and on the psychological side of Mathematics. He would suggest that the *Gazette* should devote more attention to this side of the subject. More articles might be published dealing with Mathematics from the teaching, rather than the subject-matter, point of view. Outlines of an elementary course dealing with the nature and history of Mathematics might be provided. The publication of "Occasional Papers" (apart from the *Gazette*), dealing with phases of mathematical teaching outside the text-book range, would probably be of greater value than the re-publication of "Reports" which are now out-of-date.

Again, teachers need advice in the difficult matter of text-books. A bewildering stream of elementary books—good, bad and indifferent—continues to flow unceasingly from the press. In the case of geometry especially, there is no real reason for their existence. Some of the latest are distinctly retrograde. There is nothing fresh or invigorating in them. The *Gazette* has recently dealt faithfully with one or more of them, but reviews are not always read, and the Association should not shrink from the difficult and invidious task of giving definite and decisive advice with regard to individual books.

Further, the Association should arrange for periodical informal conferences of its members, preferably at University towns during the vacations, for interchange of ideas. Teachers could learn much from the professional mathematicians and *vice versa.* The schools could get into touch with those who are training the teachers with advantage to both. Much valuable spade-work was being done by the local branches, but there should be more of them, and their proceedings should be more generally available to members of the Association elsewhere.

In the discussion which followed there appeared to be a general assent to the view that the Association is not sufficiently closely in touch with the rank-and-file of mathematical teachers in the country. Prof. Neville suggested that each number of the *Gazette* should contain a "causerie" of recent doings in the mathematical world, including records of the principal appointments: in the case of appointment to Mathematical chairs this would include a brief record of the work of the recipients. Prof. Hardy deprecated the distinction drawn by the opener between "mathematicians" and "teachers of Mathematics," contending that a great part of the work done by University Professors was directly teaching work. In reply to the Chairman, he said that he saw no reason why the suggested conferences of mathematical teachers should not be held at Oxford or Cambridge, the only difficulty being that the University teachers and lecturers might be disinclined, after an exhausting term's work, to give up part of their vacation for the purpose.

The proceedings closed with the usual votes of thanks.

REPORT OF THE YORKSHIRE (LEEDS) BRANCH FOR THE YEAR 1921.

THE branch has had quite a successful year. Many new members have been elected, including about thirty from Sheffield and district. The total number of members is now 136. Three meetings have been held—the Spring meeting on 5th March, jointly with the Leeds and District Classical Association and the Yorkshire Natural Science Association at Leeds University, when Sir T. L. Heath read a paper on "Greek Mathematics and Science," which has since been published. The Summer meeting had been arranged to take place at Hymer's College, Hull, but owing to the coal strike and curtailed train service, it was decided to hold it in Leeds on Saturday, 18th June. Mr. W. E. H. Berwick gave a paper on "Some Fallacies," in which he considered three well-known mathematical propositions which, though generally accepted as universally valid, are only so when certain conditions are presupposed. Rev. A. V. Billen also gave a paper on the question "Should Mathematics be divided into Pure and Applied in Advanced Courses in Secondary Schools ?" A discussion followed, with the result that a Sub-Committee was nominated to make proposals for the Higher School Certificate Examination of the Northern Universities, and to draw up a syllabus of combined pure and applied mathematics suitable for advanced courses in schools. The last meeting of the year was held on 3rd December at Leeds University. Professor Milne was asked to continue as president until the next annual meeting, and in place of Miss Cull, Miss Greene, and Mr. Blacklock, who were balloted off the Committee, there were elected : Miss Stephen of the Girls' High School, Leeds ; Miss Sykes of Chapel-Allerton High School ; and Dr. Brodetsky of Leeds University. Dr. Knott, Reader in Applied Mathematics in Edinburgh University, addressed the branch on "The Life and Work of Professor Tait." The Sub-Committee's report on Pure and Applied Mathematics in advanced courses was postponed until the next meeting.

SYDNEY BRANCH.

ANNUAL REPORT FOR THE YEAR 1921.

THE membership for the Sydney Branch is now : 26 members and 41 associates ; total, 67. This includes two school libraries ; and the hope is again expressed that more libraries will take the *Gazette*.

During the year three meetings were held. At the first meeting, Prof. Wellish spoke on "The Teaching of Mathematics in its Relation to Historical Development." At the same meeting a discussion took place on the proposed changes in the mathematics syllabus for the Leaving Certificate. At the second meeting two papers were read on "Courses of Mathematics as adapted for Students whose Interests will be mainly (*a*) Technical, (*b*) Commercial." These papers were presented respectively by Mr. J. B. Brown and by Mr. M. C. Alder.

At the Annual Meeting, Prof. Wellish spoke on "The Energy Principle and its Applications, with special reference to Mechanics."

The office bearers for 1922 were elected as follows : *President*, Prof. H. S. Carslaw ; *Hon. Treasurer*, Mr. C. A. Gale ; *Joint Hon.-Secretaries*, Miss F. Cohen, Mr. H. J. Meldrum.

H. J. MELDRUM (*Hon. Sec.*).

PERSONAL NOTES.

MR. E. R. THOMAS.

MR. E. R. THOMAS, the head of the Science Department of Rugby School, has been appointed as head-master of the Newcastle Grammar School. Mr. Thomas is a son of Alderman Daniel Thomas, of Aberystwyth, an old pupil of Aberystwyth County School and a former student of the University College of Wales.—[*Educational Times.*]

MATHEMATICAL NOTES.

613. [Q. 1. a.] *Closed Euclidean Spaces.*

The results of dispensing with one of Euclid's postulates are simpler if we merely dispense with that relating to our power of unlimited production of a straight line than if we dispense with the 5th postulate.

By a space we mean a manifold, and the simplest is one of discrete points, in which distances along a straight line are compared by counting point intervals. We can make the point intervals equal by definition, the dimensions of the manifold at right angles by definition, and in fact all the definitions such that its geometry is necessarily Euclidean and has no dependence on experimental results.

The only properties we need attribute to the points of the space are those which enable us to classify them as a manifold.

If the space is of two dimensions, every point of it lies on two intersecting orders; if of three dimensions, every point lies on three intersecting orders.

These orders may, however, be regarded as closed orders instead of open ones.

Every straight line then, when produced far enough, returns into itself, or follows a path like a helix, but the space is no less Euclidean than before. The sum of the angles of a triangle is two right angles; through a point only one parallel can be drawn to a given straight line; and all parallel lines have the same entire length. All Euclidean constructions with lines can be carried out, provided our lines are not too long for the space, and that we confine attention to the shortest of the straight lines between two points.

If by a "straight" line is meant an optical line, we cannot visualise a closed Euclidean space of even one dimension, much less those of two or of three, a difficulty which they share with the hypersolids of spaces of more than three dimensions.

The nearest we can get in the way of models (and those not satisfactory ones) are, firstly, the surface of a cylinder, which corresponds to a Euclidean space of two dimensions with only one dimension closed.

Secondly, the surface of an anchor ring.

We can construct a network or double order of points on the surface of the anchor ring, making the network a two-dimensional Euclidean "space" by suitable definitions.

Though the network is Euclidean, in the sense that the fifth postulate holds, its "straight" lines are of course not optically "straight," nor are equal segments on two lines necessarily equal in Euclid's meaning. Both dimensions of the network are closed orders, and in fact, from Euclid's standpoint, each point is at the intersection of two circles. The anchor ring can thus serve as a model of a closed Euclidean space of two dimensions.

The surface of a sphere is of no use to us, because it represents a Riemannian space instead of a Euclidean one, and does not lend itself to classification of its points as a manifold.

The object of the above remarks is to call attention to a departure from the ancient Euclidean geometry, which is simpler than either non-Euclidean geometry or geometry of more than three dimensions, though without their mathematical interest. And it may also be helpful to beginners in non-Euclidean geometry to correct the impression that closed spaces result only from the "hypothesis of the obtuse angle."

T. C. J. Elliott, Capt. R.A.O.C.

614. [J. 2. c.] *Put and Take.*

In this game any number of players (only two considered here) put one counter each into the pool. The players in turn spin a hexagonal top which carries the inscriptions: "Put one," "Put two," "All put," "Take one," "Take two," "Take all."

If the top comes up " Take 1, 2, or all," the spinner takes from the pool 1, 2, or the whole pool.
" " " Put 1 or 2," the spinner puts 1 or 2 into the pool.
" " " All put," each player puts one into the pool.
Whenever the number of counters in the pool becomes less than the number of players, each player puts in one.

Required to find the expectation of gain of the next spinner when there are n counters in the pool, considering only one " round," or period until the pool is cleared out, without reference to the next round. Assume only two players. W. HOPE-JONES.

615. [V. a. λ.] *The Notation of the Calculus.*

Is it not time that a serious effort should be made to bring the notation of the Calculus into line with the way it is taught ?

If we are content to regard dx as a quantity so small that its square may be neglected, it is quite natural to write $\frac{dx^2}{dx}=2x$ and $\int 2x\,dx=x^2$ (and incidentally it is easy to 'prove' these facts). But if we are to insist that $\frac{dy}{dx}$ is *not* a fraction but the limit to which a fraction $\frac{\delta y}{\delta x}$ approaches, it is most misleading to use the fractional form, and in fact beginners are, I believe, nearly always thus misled. Also, if $\int dx$ is to be thought of as *one* symbol it is most misleading to separate the $\int$ from the dx by perhaps a long bracket containing the integrand.

What are needed are signs to denote differentiation and integration which mention the variable.

The symbol Dy is often used, but this fails to mention the variable.

May I suggest that a very simple written symbol would be to write Dy and insert an x *inside* the D (I do not ask the printer to cope with this). For integration one might write D^{-1} with x inside the D or might continue the tail of $\int$ upwards so as to make a U to the left (but below the line), inside which an x might be written ; in either of these cases the dx after the integrand being omitted.

Probably some of your readers can suggest a notation more satisfactory to the printer and equally easily written.

If any such notation were adopted the *apparent* obviousness of the change of variable which replaces $\int dx$ by $\int d(x+a)$ would cease, just as a more satisfactory phrasing would prevent the statement 'an absolutely convergent series is convergent' from appearing a truism.

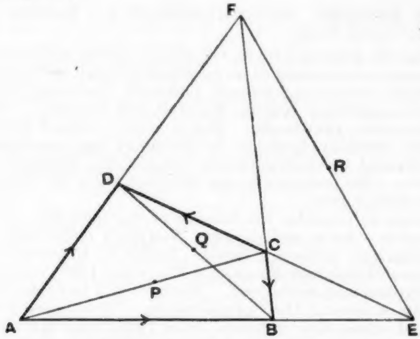
As an alternative to changing the fractional notation for $\frac{dy}{dx}$ perhaps we might (in common with most of our pupils) think openly of $\frac{dy}{dx}$ as a fraction and of dx as a quantity so small that its square may be neglected, and continue the Calculus on these lines till a point is reached where there is a danger of erroneous results being obtained, at which point a rigorous discussion of limits on modern lines would be less likely to fall upon deaf ears. That the beginning is made thus seems to me to be, even more than its racy style, the secret of the success in its aim of 'Calculus made Easy,' but I only know of one of the more complete text-books (Mr. Lodge's) which adopts this method.

C. O. TUCKEY.

616. [X. 10.] Without wishing in the least to cavil at its inclusion in the *Gazette*, I am anxious to know in what sense the solution of the *Daily Mail* puzzle on pp. 18-19 of this volume is " mathematical." E. H. NEVILLE.

617. [V. 1. a. μ.] *Vector Proof that the Mid-points P, Q, R of the Diagonals AC, BD, EF of a Complete Quadrilateral are collinear.*

Consider the system of four forces $\overline{AB}$ along AB, $\overline{CB}$ along CB, $\overline{CD}$ along CD and $\overline{AD}$ along AD.



We have $\begin{cases} \overline{AB} \text{ along } AB+\overline{AD} \text{ along } AD=2\,.\,\overline{AQ} \text{ along } AQ, \\ \overline{CB} \text{ along } CB+\overline{CD} \text{ along } CD=2\,.\,\overline{CQ} \text{ along } CQ. \end{cases}$

Also, $\quad 2\,.\,\overline{AQ}$ along $AQ+2\,.\,\overline{CQ}$ along $CQ=4\,.\,\overline{PQ}$ along PQ.

Again,

$$\begin{cases} \overline{AB} \text{ along } AB+\overline{CD} \text{ along } CD=(\overline{AC}+\overline{CB}+\overline{CA}+\overline{AD}) \text{ all through } E \\ \qquad\qquad\qquad\qquad\qquad\quad =(\overline{CB}+\overline{AD}) \text{ through } E. \end{cases}$$

Also, $\quad \overline{CB}$ along $CB+\overline{AD}$ along $AD=(\overline{CB}+\overline{AD})$ through F.

$\therefore$ the whole = a force through R.

$\therefore$ P, Q, R are collinear. W. J. DOBBS.

618. [I. 1.] *Note on Note* 610.

The paradox disappears if, after defining a surface as the aggregate of points distributed symmetrically, one proceeds to define the area of a figure as the ratio of the number of points composing it to the number of points composing the figure chosen as the unit of area.

The guile of this "Mathematical Recreation" appears to lurk in the tacit assumption that the elements of area surrounding the $\frac{1}{2}m(m+1)$ points can be *squares*, whereas they must be either hexagons or rhombi, each equal to twice the equilateral triangle determined by three neighbouring points.

The Roman formula consequently requires a multiplying factor $\sqrt{3}/2$ to convert from hexagonal or rhombic units of area to the square units commonly employed. Otherwise there will be a minimum error of approximately 13·4 per cent. L. LINES.

Oakdene, Greenfield Road, Colwyn Bay.

619. [X. 4.] *On Note* 597.

On a sheet of ruled paper draw a line perpendicular to the ruled lines, and through the point at which it cuts the top line draw two lines equally inclined to it.

Mark these lines 1, 2, 3 . . . where they cut the ruled lines and mark the perpendicular ·5, 1, 1·5, 2 . . . where it cuts the ruled lines. Then if a ruler meets the side lines at a and b and the central line at x,

$$\frac{1}{a}+\frac{1}{b}=\frac{1}{x}.$$

C. H. HARDINGHAM.

REVIEWS.

Applied Calculus. By F. F. P. Bisacre, O.B.E., M.A., B.Sc., A.M.Inst.C.E. Pp. xiv + 446. 10s. 6d. 1921. (Blackie and Son.)

Elementary Analysis. By C. M. Jessop, M.A. Pp. 174. 6s. 6d. 1921. (Cambridge University Press.)

Upon opening Mr. Bisacre's book, the reader will be surprised and delighted to find a frontispiece consisting of an excellent portrait of Newton. There are four other plates, containing sixteen portraits, including those of Kepler, Napier, Descartes, Leibnitz, Kelvin, Maxwell, and Faraday. This is following the lead of American text-books. But a really original feature is that, in addition to the usual applications to geometry and mechanics, three long chapters are devoted to electricity and magnetism, physical chemistry, and thermodynamics. No previous books, not even those of Perry or Rose, have ranged over so wide a field.

It is interesting to consider the history of the inclusion of applications in works on calculus. As is well known, Newton's definitions were given in terms of dynamical notions. Brook Taylor's *Methodus Incrementorum Directa et Inversa* (1715) and Simpson's *Fluxions* (1737) contained numerous references to physics and astronomy. Authors of a later date failed to maintain the breadth of view of their predecessors. There were a few exceptions, like Price, two of whose four volumes (from 1848) were devoted to mechanics, but their influence was comparatively small. Todhunter's books (from 1852) almost entirely excluded all applications other than geometrical. These works had a wide circulation and ran through many editions. Succeeding authors followed on somewhat similar lines. Most of them applied integration to centroids and moments of inertia, but at this their mechanics ended, and other branches of physics were hardly mentioned at all. On the other hand, higher plane curves were treated in considerable detail. For fifty years English teaching became more or less standardised on these lines. Then came a reaction. Lamb's *Calculus* (1897) made a sharp break with tradition. The admirable collection of applications to mechanics and physics was at once appreciated by teachers who might otherwise have been disposed to regard the book with some suspicion because of its introduction of unfamiliar continental ideas on limits. Gibson (1901) pursued the same policy, relegating higher plane curves to a minor position but treating limits with great care, and illustrating differentiation by examples dealing with dynamics, electricity, and thermodynamics. In 1897 Perry brought out his *Calculus for Engineers*. This required an extensive knowledge of mechanical and electrical engineering, and so was not suitable for the ordinary mathematical student, but it was greatly appreciated by those for whom it was intended. Rose's *Mathematics for Engineers* (vol. i. 1918, vol. ii. 1920) is a recent work of a similar type. Turning to continental writers such as Jordan, Picard, Goursat, and De La Vallée Poussin, we find scarcely any references to mechanics or physics. The superiority of these books on theoretical points is universally acknowledged, but their influence on the English reader would be greater if the admirable chains of abstract reasoning received some physical application. Possibly English and continental minds work in different ways, but it is significant that it has been found necessary to translate Perry's *Calculus for Engineers* into French and German.

Of course the inclusion of applications has disadvantages as well as advantages. If a knowledge of thermodynamics is assumed when partial differentiation is explained, the discussion is unintelligible to those who do not know this branch of physics. If, on the other hand, the thermodynamics is fully explained, the explanation becomes so long that the partial differentiation is obscured. The best way of avoiding this dilemma is to state a theorem in terms of pure mathematics in the first place, and then to add a note giving briefly the physical meaning of the symbols used and of the result proved, with a reference to a definite page of some standard text-book on physics where the matter is discussed exhaustively. However, Mr. Bisacre has tried to explain most of his physics at full length. He states that his book is intended

for students of science and engineering, whose knowledge of mathematics need not extend beyond quadratic equations, the trigonometrical addition formulae, elementary geometry, and graphs. Unfortunately the amount of science and engineering supposed known is not explicitly stated. It seems to be considerable, for the following are used without explanation: Newton's Laws of Motion applied to a rotating rigid body, dyne, electromotive force, hysteresis, coulomb, r.p.m., molecular weight, molecular equation, atomic hydrogen, dissociation, tensile stress, B.Th.U., and K.W.H. Many scientific terms are carefully defined, but as there is no index it is not always easy to find these definitions, in spite of the fairly full table of contents.

Many teachers of engineering students imagine that a picturesque inaccuracy of language makes a theorem more acceptable to their classes. Mr. Bisacre is not one of these. His language can be as picturesque as anyone's, as for example in his illustration of the idea of a limit by Portia's speech from *The Merchant of Venice*, but he has taken great pains to be accurate at the same time—the definition of *function* on p. 24 is rather loosely worded, but this is an isolated slip. In the differentiation of standard forms the use of infinite series has been carefully avoided.

Probably enough has been said to convince every teacher of calculus that he should obtain a copy of this highly original book, but there remains the question whether it is suitable for class use. Unfortunately there are some reasons for fearing that it may prove unworkable as a text-book. The careful and witty explanations and the wealth of applications are delightful to those who are familiar with the leading ideas of the subject, but they are likely to bewilder a beginner. The rapid alternations between different topics are confusing. Theorems are presented in massed formation (see, for instance, Chap. VI.), insufficiently elucidated by examples. The hints for the solution of the miscellaneous examples at the end of the book partly atone for this defect, but it is difficult to believe that students restricted to the examples given by Mr. Bisacre will be able to perform with confidence the routine processes of differentiation and integration needed in science and engineering.

We now turn to Prof. Jessop's book. The amount of mathematics presupposed is about the same as for Mr. Bisacre's. In other respects the works are in strong contrast. Prof. Jessop is, above all, concise. In less than two hundred pages he provides a first year course in differential and integral calculus and coordinate geometry, with a good stock of well-chosen examples which break up the bookwork into small portions that can be easily assimilated. The last chapter is rather fragmentary. It is entitled *The General Conic*, but it also includes a treatment of the parabola, ellipse, and hyperbola referred to their usual axes. The applications of the calculus are restricted to geometry and mechanics. The mechanics is introduced briefly but clearly, requiring no previous knowledge of this subject.

Theorems concerning limits, to which Mr. Bisacre devotes about forty pages, are dealt with by Prof. Jessop in ten. Probably the junior student will prefer the shorter treatment, but the mathematical specialist may complain that from this treatment the idea of *limit of a function of a continuous variable* has been omitted. The explanation of *limit of a sequence* is certainly given, but it is the former that is required in the definition of a differential coefficient. This point is slurred over by a misuse of the word *sequence*. Another matter that should be altered in the next edition is the statement at the top of p. 83, that at a minimum d^2y/dx^2 is positive. This is not always true, as may be seen by considering the curve $y=x^4$ at the origin, where y is a minimum but d^2y/dx^2 is zero. Of course it is correct to say that if dy/dx is zero and d^2y/dx^2 is positive, then y is a minimum. As is usual, but regrettable, both Mr. Bisacre and Prof. Jessop ignore the simple problem of determining the *greatest* value of a function when the independent variable is confined to a *restricted* range. This may occur in determining at what point of a finite rod the tendency to break is greatest.

Prof. Jessop's method of dealing with logarithmic and exponential functions begins by showing that the differential coefficient of $\log_a x$ is the limit, when z approaches zero, of $\frac{1}{x}\log_a\{(1+z)^{\frac{1}{z}}\}$. This brings us up against the difficulty of the exponential limit. For a discussion of it the student is referred to

Hardy's *Pure Mathematics*. As soon as this limit is obtained all goes smoothly. The result $d(\log_a x)/dx=(1/x)\log_a e$ shows the great advantage of taking e as the base of our logarithms. The exponential function is then differentiated by treating it as the inverse of the logarithmic function. This treatment, like all Prof. Jessop's work, is remarkably concise, for the theory, together with graphs and examples, occupies less than three pages of clear type.

Mr. Bisacre defines the exponential function of x as the number obtained by raising a certain irrational number $e=2{\cdot}71828...$ to the power x, for real values of x. Nine pages later he admits that the reader may still be at a loss as to why this curious number is used, but he gives it to be understood that if any other number had been chosen the derivative of the exponential function would not have come equal to the function itself. This treatment seems less natural than that of Prof. Jessop.

To conclude, Mr. Bisacre's book is one whose originality and wide range of applications should make it a welcome addition to any teacher's library, while Prof. Jessop's is a concise and workable course for junior college students.

H. T. H. Piaggio.

The "Wee" Mathematical Series. (Bowman and Murdoch.)

Arithmetical Calculating Slips. These are Napier's bones, with stout cardboard (1s.) replacing the wood or metal of a less democratic age. A set consists of one slip for each digit, and therefore a single set is inadequate if a multiplicand or a dividend contains any integer more than once. The instructions will convey little to anyone who needs instructions at all, but since "the principle on which this Calculator is based is fully explained in the *Simplified Arithmetic*, which will be ready shortly," reference to seventeenth century explanations would be an impertinence on our part. We venture to doubt whether by the use of these detached columns "a grasp of the significance of numbers, almost impossible in any other way, can be obtained," but they could be introduced into the teaching of the multiplication table, and if they were dissected further, an instructive variety of "Happy Families" could be played with the elementary squares.

The "Wee" Table of Logarithms (3s. 6d.) is mounted on rollers, after the fashion of a perpetual calendar, to prevent the eye from straying into the wrong line or column. The device is clumsy, for a straight-edge marked with the column-headings would be simpler and far quicker to use. No acknowledgment is made, but the table seems to be that of which Messrs. Macmillan hold the copyright. E. H. N.

Four-Figure Logarithms and other Tables. By G. Hogben. New edition, undated. 2s. 6d. (Whitcombe and Tombs, New Zealand.)

These excellent tables cover the usual ground, but by a liberal use of split lines the errors in the fourth place of decimals are unusually small. It follows that three-figure accuracy is secured for chains of operations so long that on the ordinary plan five-figure tables would be necessary to give confidence. Paper and print are good, and we have nothing against the tables but their price, which, by English standards even at the present time, is high. E. H. N.

First Principles of Jig and Tool Design. By F. Lord. Pp. 69 + Charts A-X. 3s. 6d. net. 1921. (Blackie.)

In this hand-book the author purposes to describe the nature of jigs and tools, their design and how they are made, for the information of young engineering students. It is well thought out, and profusely illustrated, with a good bibliography and a list of most useful charts of standard fittings, tools and gauging systems.

Altogether it is a most useful book, although it is a pity the author has not made some passing reference in his chapter on Gauges to the Johansen system, for accurate checking, and the Prestwich Fluid Gauge for rapid checking on mass production work. Also with advantage the text on the manufacture of jigs could have contained a description of the location of centres by means of ground cylinders or buttons, the sine bar and micrometers from some datum surface.

Pattern Making. By W. R. NEEDHAM. Pp. v + 114. 2s. 6d. net. 1921. (Blackie.)

This book, with a companion volume on Foundry Practice, should prove a great boon to young apprentices. The treatment, necessarily of an elementary nature, is simple but sound, the author never allowing the close coordination of pattern making and moulding to be forgotten.

It is well illustrated and covers a wide range of work, including the hand and machine tools, the manufacture of metal patterns and multiple production on the plate principle.

The author could with advantage, in his chapter on Struck Work and Sweeps, have elaborated somewhat on skeleton pattern work for the construction of pump volutes, turbine exhaust casings, and patterns of similar construction, which may be easily made up by the principles of generative geometry or methods of radial section. F. C. MUSTARD.

Elementary Algebra. PART II. By C. V. DURELL and R. M. WRIGHT. Pp. xxiii + 253-556 + xlvii-lxxxv. 5s. 6d.; without Introduction and with select Answers, 4s. 6d. 1921. (Bell & Sons.)

Of the general merits of Part II. we can but repeat what has already been said of Part I. in the *Gazette*. It remains to indicate the ground covered. The book opens with chapters on indices, logarithms, ratio, proportion, and variation. A chapter on functions of one variable provides the general characteristics of simple functions. Next come sections on the ideas of limits and gradients, and then follow chapters on differentiation, and on integration as the reverse process of differentiation. Simple accounts are given of Dufton's and Simpson's approximate rules. Elementary series are now treated in the simplest possible way, the point of view being the "definite value in the appreciation of any kind of algebraic form, and in the cultivation of the power to generalise ideas and to utilise formulae. If the reader can be led to construct formulae for himself and to understand the idea of order, he will gain definite advantage from this chapter." From Permutations and Combinations, with a few remarks and examples on easy Probabilities, the student is led to the Binomial Theorem and its application to approximate calculations. A very useful chapter on empirical formulae precedes a chapter on Nomography, with a series of carefully worked out examples. So far the greater part of the book is for the non-specialist. The final pages deal with the theory of quadratics, Algebraic form, Σ notation and the like. About eighty pages are given to Revision Exercises, and the book closes with a Glossary, Index and log. tables.

Obituary Notices.

MR. H. G. MAYO.

WE regret to notice the death of Mr. H. G. Mayo, M.A. Cambridge, B.Sc. London, and L.C.P., who died on 27th December, 1921. He was educated at the Marling School, Stroud, and was a Scholar of Queen's College, Cambridge. He was successively Assistant and Second Master of Norwich Middle and Norwich City Schools, respectively, 1905-1913, and was Head Master of Ossett Grammar School from 1913-1917. Leaving Ossett, he was Assistant Master at Bristol Grammar School, 1917-1920. He was then appointed as the first Head Master of the new Oldershaw Secondary School. At the time of his death he was working with Prof. Tyndall, of Bristol University, on a new form of Hygrometer, for use in cold storage. His loss will be lamented by a large circle of pupils and friends.

MR. C. H. HODGES.

THE death is reported in New South Wales of Mr. C. H. Hodges, formerly assistant master in Rugby School, where he is remembered as a sportsman and a teacher of rare distinction. Mr. Hodges was educated at Carlisle School and Queen's College, Oxford, where he took a first in the mathematical final school.

After some experience as an assistant master at Radley, he was appointed on the staff at Rugby, but a breakdown in health caused him to seek a more congenial climate. He settled in Queensland as head master of Townsville School, and later became head master of Sydney School, which he developed into the best equipped and most important school in the Commonwealth. Mr. Hodges was regarded as one of the great constructive head masters and the pioneer of the public-school system in Australia.

Mr. Reginald Saumarez de Havilland.

Etonians all over the world will receive the news of the death of Mr. Reginald Saumarez de Havilland with grief. Mr. Havilland was one of the best, if not the best, of rowing coaches the school ever had, and Eton's many successes in the Ladies' Plate at Henley during the past decade were largely due to his skill. After a distinguished career at Eton he went up to Oxford and obtained a third in the Mathematical Final School in 1884. He rowed in the Oxford Eight and became President of the O.U.B.C. He returned to Eton as an assistant master in 1889, and retired in 1920, when Mr. C. J. M. Adie took over his house. He was appointed Major Commandant of the Eton College Rifle Volunteer Corps in 1905. A successful house master and a most faithful friend, his cheery disposition early gained him the title of "Happy," afterwards altered to "Havi." Beloved by all with whom he came into contact, his memory will be cherished with affection by many generations of Etonians.—[*Journal of Education.*]

ENQUIRY.

Prof. W. H. Young, F.R.S., University College of Wales, Aberystwith, will be glad to hear of isolated volumes or sets of the *Math. Annalen* for sale.

THE LIBRARY.

The Library has now been removed to 29 Gordon Square, London, W.C. 1 and Mr. W. E. Paterson has taken over the duties of Honorary Librarian.

The Librarian will gladly receive and acknowledge in the *Gazette* any donation of ancient or modern works on mathematical subjects.

Regulations for the Use of the Library by Members.

1. Any member of the Association is entitled to borrow books from the Library (except those marked in the catalogue with an asterisk).
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3. The borrower must pay carriage both ways, and will be held responsible for any loss or damage.
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Scarce Back Numbers.

Reserves are kept of A.I.G.T. Reports and Gazettes, and, from time to time, orders come for sets of these. We are now unable to fulfil such orders for want of certain back numbers, which the Librarian will be glad to buy from any member who can spare them, or to exchange other back numbers for them :

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A.I.G.T. Report No. 11 (very important).
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THE MATHEMATICAL ASSOCIATION.

(An Association of Teachers and Students of Elementary Mathematics.)

"I hold every man a debtor to his profession, from the which as men of course do seek to receive countenance and profit, so ought they of duty to endeavour themselves by way of amends to be a help and an ornament thereunto."—Bacon.

The Mathematical Association, which was founded in 1871, as the *Association for the Improvement of Geometrical Teaching*, aims not only at the promotion of its original object, but at bringing within its purview all branches of elementary mathematics.

Its purpose is to form a strong combination of all persons who are interested in promoting good methods of teaching mathematics. The Association has already been largely successful in this direction. It has become a recognised authority in its own department, and has exerted an important influence on methods of examination.

The Annual Meeting of the Association is held in January. Other Meetings are held when desired. At these Meetings papers on elementary mathematics are read and discussed.

Branches of the Association have been formed in London, Southampton, Bangor, and Sydney (New South Wales). Further information concerning these branches can be obtained from the Honorary Secretaries of the Association.

"The Mathematical Gazette" (published by Messrs. G. Bell & Sons, Ltd.) is the organ of the Association. It is issued at least six times a year. The price per copy (to non-members) is usually 2s. 6d. each. The *Gazette* contains—

(1) Articles, mainly on subjects within the scope of elementary mathematics;

(2) Notes, generally with reference to shorter and more elegant methods than those in current text-books;

(3) Reviews, written by men of eminence in the subject of which they treat. They deal with the more important English and Foreign publications, and their aim, where possible, is to dwell on the general development of the subject, as well as upon the part played therein by the book under notice;

(4) Short Notices of books not specially dealt with in the Reviews;

(5) Queries and Answers, on mathematical topics of a general character.

Intending members are requested to communicate with one of the Secretaries. The subscription to the Association is 15s. per annum, and is due on Jan. 1st. It includes the subscription to "The Mathematical Gazette."